PROCEEDINGS OF THE MATHEMATICS IN INDUSTRY STUDY GROUP

2019

Mathematics in Industry Study Group South Africa
MISGSA 2019

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The sixteenth Mathematics in Industry Study Group (MISG) in South Africa was held in theAfrican Institute for Mathematical Sciences, Muizenberg, Cape Town, from Monday 14 January to Friday 18 January 2019.

The total number of registered participants at the MISG was sixty-three. There were twenty Academic Staff, thirty-six Graduate Students and seven Industry Representatives. The invited guests were:

Philip Broadbridge	La Trobe University, Melbourne, Australia
Neville Fowkes	University of Western Australia, Australia
Graeme Hocking	Murdoch University, Western Australia, Australia
Tim Myers	Centre de Recerca Matematica, Barcelona, Spain
Denis Ndanguza	University of Rwanda, Rwanda
Colin Please	University of Oxford, United Kingdom

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Sciences
Durban University of Technology
North-West University
University of Cape Town
University of Johannesburg
University of KwaZulu-Natal
University of Pretoria
University of South Africa (UNISA)
University of Stellenbosch
University of Venda
University of the Witwatersrand
University of Zululand

The MISG was officially opened on Monday morning by Professor Barry Green, Director of the African Institute for Mathematical Sciences.

The MISG followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2018. Eight problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by an academic moderator and one or more student moderators. The role of the academic moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results which were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Modelling Camp from Wednesday 9 January to Saturday 12 January 2019. The objective of the Graduate Modelling Camp is to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working collaboratively in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. Six problems were presented to the graduate students. The problems and the presenters were:

Mathematical modelling of rogue waves	Thama Duba Durban University of Technology
Decision support tool for optimal beer blending	Matthews Sejeso University of the Witwatersrand

Laser techniques for deter– mining the conductivity of surface films	Neville Fowkes University of Western Australia
On-line auction	Jeff Sanders African Institute for Mathematical Sciences and University of Stellenbosch
A start-up game farm	Ashleigh Hutchinsom University of the Witwatersrnad

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

The sponsors of the Graduate Workshop and the MISG were:

- Hermann Ohlthaver Trust
- African Institute for Mathematical Sciences
- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences
- School of Computer Science and Applied Mathematics, University of the Witwatersrand

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

STUDY GROUP

Participants

Academic staff	
Adeleke, Olawale	University of the Witwatersrand
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Fredericks, Ebrahim	University of Cape Town
Hale, Nick	University of Stellenbosch
Hocking, Graeme	Murdoch University, Perth, Australia
Hutchinson, Ashleigh	University of the Witwatersrand
Khalique, Masood	North-West University
Mason, David	University of the Witwatersrand
Matadi,Boniface	University of Zululand
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Please. Colin	University of Oxford, United Kingdom
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Wolway, Matthew	University of the Witwatersrand

Graduate Students	
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Atherfold, John	University of the Witwatersrand
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Bright, Chelsea	University of Pretoria
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Tilahun, Surafel	University of Zululand
Uzor, God'sgift	University of the Witwatersrand
Industry Representatives	
Broadbridge, Philip	La Trobe University, Melbourne, Australia
Duba, Thma	Department of Mathematics, Durban University of Technology
Holtz, Norman	aparate
Louber, Richard	Sugar Milling Research Institute, University of KwaZulu-Natal, Durban
Myers, Tim	Centre de Recerca Matematica, Barcelona, Spain
Ndanguza, Denis	Department of Mathematics, College of Science and Technology, University of Rwanda, Rwanda
Van Zyl, Terence	School of Computer Science and Applied Mathematics, University of the Witwatersrand

GRADUATE MODELLING CAMP

Participants

Coordinator	
Hutchinson, Ashleigh	University of the Witwatersrand
Problem presenters	
Duba, Thama	Durban University of Technology
Fowkes, Neville	University of Western Australia
Huchunson, Ashleigh	University of the Witwatersrand
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Sanders, Jeff	AIMS and University of Stellenbosch
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Tilahun, Surafel	University of Zululand
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Nkolele, Risuna	University of the Witwatersrand
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Seota, Bill	University of the Witwatersrand

Sepuru, Shaun	University of the Witwatersrand
Thulare, Evans	University of the Witwatersrand
Uzor, God'sgift	University of the Witwatersrand

PROBLEM STATEMENTS

Problem 1. Visual animal

Industry: Wildlife management

Industry Representative: Terence van Zyl, University of the Witwatersrand, Johannesburg

Problem statement:

Visual animal is an emerging research discipline in computer vision, pattern recognition and cognitive science. Many animals may be recognized by individual biometric characteristics in just the same way as human fingerprints. Since human fingerprints are relatively small and hidden characteristics, they cannot be used by other persons for mutual visual identification. In contrast to human fingerprints, there are animal skin patterns which are large enough to be used for mutual recognition without technical aids - at least theoretically.

Here we wish to given a number of examples of say for instance gazelle be able to uniquely identify each of them. This is very useful for game/livestock tracking and counting in nature conservationist settings.

Problem 2. Optimizing urban and roof-top farming in Soweto and South Johannesburg

Industry: Agriculture

Industry Representative: Norman Höltz, aparate

Problem Statement

Urban and roof-top farming is important for food security in South Africa. Entrepreneurial and employment opportunities in urban farming have significantly risen in the past few months in Soweto and South Johannesburg. In contrast to traditional farming, urban and roof-top farming yields small crop sizes and are spread out throughout the city. Roof-top farming can be done year round as it is farmed hydroponically under controlled conditions. Hydroponic farming requires no soil, instead the roots are suspended in mineral rich static or continuously moving water. Ultraviolet lamps provide continuous artificial sunlight. On roof-tops, plants are grown in greenhouses to protect them from extreme temperatures, wind and pests. Farms can now produce food throughout the year.

The produce of the farms has to be collected and transported to the market. With urban farming, there are a large number of smaller farms in contrast to the norm of one or two large farms. These smaller farms are also separated by some distance.

Since the farming can be done throughout the year, the study group is asked to optimise the plant and harvest times for a large number of farms producing a range of products. Prices at different times of the year in South Africa and growing times for the produce will be provided.

The study group is also asked to optimise the routes, by road, between the farms during the collection of produce. Some produce, for example strawberries, require refrigeration which not all vehicles have. The optimal routes for non-refrigerated vehicles will therefore differ from that for refrigerated vehicles. The locations of the farms will be provided. The study group has to determine how the routes are calculated, for example, with Google maps.

Problem 3. Rogue waves

Industry: Shipping and Offshore Oil and Gas

Industry Representative: Thama Duba, Durban University of Technology

Problem Statement:

Rogue waves are waves whose height and crest exceed the average wave in a given sea state. These abnormal waves have received significant attention both in media and in the scientific community, especially in the shipping and offshore industry. Rogue wave accidents involving large ships and offshore structures are on the rise as these waves are no longer rare. In 1995, a 23-meter high wall of water struck the Draupner platform and this caught the attention of oceanographers, and the oil and gas and other offshore industries. Abnormal waves no longer became a sailor's folklore but a problem to these industries.

Rogue waves have an impact on standard shipping and offshore rules. Rogue waves occur in different sea states, low, intermediate and high sea states. Rough sea states are no longer the only ones that we must take into account in the operation of ships, but all sea-states due to these monster waves. Therefore, it is imperative that we develop new warning criteria for the rogue-prone sea-states. Furthermore, the engineering design of ships, offshore structures and platform decks will have to be different as rogue waves are no longer rare, probably due to climate change. In addition, it is not clear when they will appear, hence under such uncertainties, it is necessary to have new regulations for safety standards.

The study group is asked to investigate the initial conditions of the sea state from which rogue waves can form and grow.

Problem 4. Sugar crystal counting, size and quality analysis

Industry: Sugar Cane Processing

Industrial Representative: Richard Loubser, Sugar Milling Research Institute, c/o University of KwaZulu-Natal, Durban.

Problem Statement

Under ideal conditions, the sugar crystals produced in the sugar process need to be uniform size and have a regular shape. This simplifies the design of equipment such as centrifuge screens. In reality, however, a range of crystal size is produced and impurities cause crystals to elongate resulting in a poor crystal habit.

Photomicrographs are used to manually assess massecuites (mixture of crystals and mother liquor) samples in terms of mean size (length and width), median (length and width), size coefficient of variation (CV) and elongation ratio (length to width ratio). In addition to this, it is often necessary to estimate the number of crystals in a unit volume of syrup (in the order of 10^5 crystals per mL).

In certain circumstances, the image may include air bubbles, agglomerated crystals, conglomerates and crystals which touch or overlap. These must be excluded from the analysis. However, the conglomerates percentage by number is of importance w.r.t sugar quality and will need to be counted.

We are looking for an algorithm that will identify single crystals, count them and give size, distribution and shape statistics.

Several photomicrographs will be made available for development and testing.

Problem 5. Lake Kivu surface water pollution

Industry: Energy

Industry Representative: Denis Ndanguza, University of Rwanda, Rwanda

Problem Statement:
Lake Kivu is known to be a dangerous lake in East Africa. A new type of hazard which may occur is now to be taken seriously under consideration in relationship with the existence of high concentration of dissolved gas in the water at depth in the lake. This new hazard is the water pollution and it seems that living around Lake Kivu means accepting a higher risk than living elsewhere in the region.

The most obvious type of water pollution affects surface water. Most water pollution does not begin in the water itself. The literature argues that there is a quantity of gas vented in the atmosphere during the harvest and there are other toxic substances entering the Lake; all those substances get dissolved or lie suspended in water. Chemicals released by smokestacks (chimneys) can enter the atmosphere and then fall back to earth as rain, entering the Lake and causing water pollution. This results in the pollution of water. The pollution may cause the quality of the water to deteriorate and further affects the aquatic ecosystems. These pollutants can also seep down and affect the groundwater deposits or disturb density-stratified lake waters.

In general, water pollution has many different causes and this is one of the reasons why it is such an interesting problem to solve. There is a clear danger lying ahead, and something needs to be done in advance to alert decision makers from all points of view (environmentally, economically and risk monitoring). To do nothing would be completely unacceptable.

In the course of the methane extraction by venting a certain percentage of gas in the atmosphere, it would increase the risk of water pollution, which is why the continuous monitoring and regulation is mandatory. Therefore, the extraction projects have to be regulated and well monitored.

The Study Group is asked to model the surface water pollution in Lake Kivu.

Problem 6. The double diffusive convection effect in Lake Kivu

Industry: Energy

Industry Representative: Denis Ndanguza, University of Rwanda, Rwanda

Problem Statement:

Lake Kivu is a stratified lake with several gradient layers (density variation of the water with depth) which serve as ``flexible lids'' ensuring both a resistance to mixing (which could cause a gas release), and a barrier which allows for the accumulation of methane gas (and carbon dioxide) in the lake. All four layers have different characteristics and composition. Notably there are two gradient layers at about 80 and 260 m depth, respectively, where the upper layer protects the overlying biozone and the lower layer confines and protects the major part of the gas deposit. The gas is kept dissolved at the lake bottom by the weight of water containing nutrients above it, and mixing toward the surface is prevented by strong density gradients which act like stability layers at certain depths in the lake. This may result in double diffusive convection where two different density gradients have different rates of diffusion.

Convection in fluids is driven by density variations within them under the influence of gravity. These density variations may be caused by gradients in the composition of the fluid, or by differences in temperature (through thermal expansion). Thermal and compositional gradients can often diffuse with time, reducing their ability to drive the convection and requiring that gradients in other regions of the flow exist in order for convection to continue. A common example of double diffusive convection is in Lake Kivu, where temperature and salinity or CO2 and CH4 diffuse at differing rates. Double diffusive convection plays a significant role in upwelling of nutrients and vertical transport of heat and salt in the lake.

This is opposing the thermal diffusion which is an effect wherein a temperature gradient in a gaseous or liquid mixture tends to cause a separation: the concentration of components in the regions of increased and decreased temperatures, respectively, becomes different. Since the establishment of a concentration gradient causes, in turn, ordinary diffusion, in a stationary non-uniform temperature field a steady state inhomogeneous state is possible in which the separation effect of thermal diffusion is balanced by the counteraction of concentration diffusion.

The magnitude of vertical mixing with double diffusive convection depends upon the density ratio. This ratio requires knowledge of the expansion coefficients of temperature, salinity, CO2 and CH4 as well as the change in these constituents with depth. Any change in the horizontal distribution of temperature, salinity or the dissolved gases will set the stage for an increase in intrusions formed by double diffusive convection. These will increase both lateral and horizontal mixing rates. The extraction of water at depth will create horizontal heterogeneity of properties and therefore, the following points are crucial to understanding the concept of double diffusive convection and some quantities have to be calculated.

- Due to the inferred double diffusive mixing above and below the main stability layer at 260 m and its drawdown and subsequent exposure to potentially different rates of geothermal heating which could lead to instabilities, it is imperative that this ratio be frequently calculated.
- Furthermore, given the vital role of mixing in control of lake heat storage, climate, carbon dioxide absorption and pollutant dispersal, it is increasingly important that we achieve a more complete understanding of the lake double diffusion.
- Evaluate the impacts of the heterogeneity along-side the horizontal diffusivities.
- Show how opposing stratifications of two component species could drive convection if their diffusivities differ.
- To compute the rates of mixing.
- To compute the Rayleigh number in the layers.
- To initiate the theoretical study of double-diffusive phenomena in Lake Kivu.
- To check if the lake is strongly unstable to double-diffusive processes and seems to be profoundly affected by their presence.
- To investigate the nature of double-diffusive mixing in the large, under-explored fuid which covers most of the lake
- Investigate the double diffusion effect due to the opposing effects of thermal diffusion and mass diffusion

Problem 7. Carbon capture and storage

Industry: Conservation

Industry Representative: Tim Myers, Centre de Recerc Matematica, Barcelona, Spain

Problem statement

It is generally accepted that global warming and climate change are real. The main cause of warming is the greenhouse effect, whereby the atmosphere traps heat that would otherwise radiate back into space. In reality what is really happening is that energy is being added to the system (the planet), which leads to more energetic events including global warming, but also increased likelihood of extreme events: more powerful storms; heat waves; flooding etc.

One of the worst culprits for climate change is the burning of fossil fuels, such as coal and oil. This has greatly increased the amount of CO_2 (a greenhouse gas) in the atmosphere.

Obviously one way to reduce CO_2 production is to stop the use of fossil fuels, however it seems unlikely that this will occur in the near future. Further, it is now becoming increasingly clear that it will be necessary to reduce the CO_2 already in the atmosphere if current goals on global temperature increases are to be met.

A controversial technology is Carbon Capture (CC). The most obvious form is CC technology applied close to a source, such as a power plant. Direct Air Capture (DAC) processes atmospheric air to remove CO_2. Controversy over these techniques include the cost, the fact that some countries may use this as a way to keep using ‘dirty fuels’, and what to do with the carbon or carbon dioxide afterwards.

The aim of this project is to investigate these technologies, to verify whether they are indeed viable and identify areas where mathematical modelling can help.

Problem 8. Exclusion zone for a fish population

Industry: Fishing

Industry Representative: Philip Broadbridge, La Trobe University, Melbourne, Australia

Problem statement

Recently an exact solution was produced of the Verhulst nonlinear reaction-diffusion equation for a fish population with logistic Fisher-type source term. It was assumed that the population is contained within a circular protection zone, with severe culling at the boundary. In order for there not to be a solution that approaches extinction, there is a minimum diameter of the exclusion zone. For example, for a species randomally spreading over 100 square km per year, with a population renewal time of 5 years, the minimum diameter is 103 km. How does this estimate change if we soften the boundary conditions, change to a heterogeneous habitat so that population growth is stronger in the centre, or have a 2-species ecosystem with two different mobility levels?